REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
22-02-2011

2. REPORT TYPE
Final Performance Report

3. DATES COVERED *(From - To)*
28 FEB 2007 – 31 AUG 2010

4. TITLE AND SUBTITLE
Towards Development of Nonparametric System Identification Base
Based on Slow-Flow Dynamics, with Application to Damage Detection and Uncertainty Quantification

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-07-1-0335

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR
875 N. Randolph St.
Room 3112
Arlington, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT
A theoretical framework for a new nonlinear system identification (NSI) method was developed. An equivalence between analytical slow-flows of the dynamics, derived from complexification and averaging, and empirical slow-flows, obtained directly from data as the intrinsic mode functions resulting from empirical mode decomposition, was rigorously demonstrated. The NSI method was then formulated based on multiscale dynamic partitions and direct analysis of measured time series, with no presumptions regarding the type and strength of the system nonlinearity. In fact, the method is applicable to time-variant/time-invariant, linear/nonlinear, and smooth/non-smooth dynamical systems. The method systematically leads to reduced order models of strongly nonlinear transitions in the form of coupled or uncoupled oscillators with time-varying or time-invariant coefficients forced by nonhomogeneous terms representing nonlinear modal interactions. The method identifies not only the dominant time scales of the dynamics but also the nonlinear interactions across the scales of the dynamics.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF: None			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** Lawrence A. Bergman
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			**19b. TELEPHONE NUMBER** *(include area code)* 217-333-4970

Final Performance Report to AFOSR

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TOWARDS DEVELOPMENT OF NONPARAMETRIC SYSTEM IDENTIFICATION BASED ON SLOW-FLOW DYNAMICS, WITH APPLICATION TO DAMAGE DETECTION AND UNCERTAINTY QUANTIFICATION

Grant Number: FA9550-07-1-0335
Grant Period: 28 FEB 2007 – 31 AUG 2010
Grant Monitor: Dr. David Stargel

Executive Summary

The need for system identification and reduced order modeling arises from the fact that, presented with sensor data, the analyst is generally unaware of many details of the underlying dynamical system from which they originated. The straightforward approach to this dilemma is to assume linearity and temporal stationarity in the response, and perform experimental modal analysis. This approach has served the dynamics and controls community well, even in the presence of weakly nonlinear system behavior. Clearly, though, as systems become more complex, the likelihood exists that the underlying dynamical behavior will be strongly nonlinear and nonstationary (e.g., local buckling, plastic deformations, clearance and backlash, hysteresis, etc.). Moreover, nonlinear systems routinely possess co-existing qualitatively distinct dynamics, so their responses are largely dictated by initial and forcing conditions. These well-recognized, highly individualistic features of nonlinear systems restrict the unifying dynamical features that are amenable to system identification. To address these issues we have developed a new nonlinear system identification (NSI) methodology of broad applicability, relying solely upon direct time series measurement and post-processing, and leading to *dual global / local identification* of the dynamics. Key to our method are slow/fast partitions of the measured time series, in order to identify dominant *fast frequencies* (which also define dimensionality), and derive associated *slow flows*. We employ powerful post-processing computational algorithms (wavelet transforms, Hilbert transforms and empirical mode decompositions) in order to identify the dynamics in the frequency-energy domain and study *global* features, such as fundamental and subharmonic resonances, bifurcations, multi-frequency transitions, etc. Associated with this global identification is the derivation of *local* reduced order models that model specific damped nonlinear transitions. We show that our method is especially suited for strongly nonlinear systems with high sensitivity to initial conditions and forcing giving rise to these phenomena.

Significant Work Accomplished

The difficulty in developing system identification methodologies that are valid for broad classes of dynamical systems is due to the well-recognized highly individualistic nature of nonlinear systems which restricts the unifying dynamical features that are amenable to system identification. Our technique relies solely on direct time series measurement and post processing, and leads to *global* as well as *local* nonlinear system identification (NSI) of a broad class of dynamical systems. Key to our method is the slow/fast partition of time series measurements which leads to the identification of the dominant *fast frequencies* in the measured time series (which ultimately govern the dimensionality of the dynamics), to *identification of global features of the dynamics in the frequency – energy domain*, and to *reduced slow flow models*. We employ advanced post processing computational algorithms, namely, *wavelet transform (WT)*, *Hilbert transform (HT)* and *empirical mode decomposition (EMD)* unified by a solid theoretical framework. Our technique successfully integrates analytical concepts and computational algorithms in a synergistic methodology. We now proceed to discuss in detail the basic elements of our new NSI methodology.

Reduced Slow Flow Dynamics and Empirical Mode Decomposition (EMD)

Slow-flow reduction of the dynamics is a useful tool for understanding the major features of a dynamical system. The reduced slow-flow model of a dynamical process is derived by introducing a slow/fast partition of the dynamics whereby the (non-essential) fast dynamics is averaged out to reveal the (essential) slow-flow modulations of appropriately defined amplitudes and phases. Perturbation methods have been developed to perform this task; among these the complexification-averaging (CX-A) technique is unique in its capacity to provide slow-flow models even for strongly nonlinear transient dynamical interactions; for example of resonance capture phenomena in coupled oscillators with essentially nonlinearities.

Focusing on the CX-A method, we demonstrate the extraction of the slow-flow dynamics of a general *n*-degree-of-freedom (DOF) nonlinear dynamical system of the form

$$\dot{\underline{X}} = \underline{f}\left(\underline{X}, t\right), \quad \underline{X} = \left\{\underline{x}^T \ \dot{\underline{x}}^T\right\}^T \in R^{2n}, \quad t \in R \tag{1}$$

where $\underline{x}$ is an $n-$response vector and $\underline{f}$ is an $n-$vector function (underlines denote vectors). Assume that the dynamics possesses N distinct components at frequencies $\omega_1, ..., \omega_N$, so the response of each DOF of the system can be expressed as a summation of N independent components,

$$x_k(t) = x_k^{(1)}(t) + ... + x_k^{(N)}(t), \quad k = 1, ..., n \tag{2}$$

where $x_k^{(m)}(t)$ indicates the response of the $k-th$ coordinate of (1), associated with the basic frequency ω_m with the ordering $\omega_1 > ... > \omega_N$. We assume at this point that all the basic frequencies in the response are well separated. Utilizing (CX-A), for each component in (2) we assign a new complex variable defined by

$$\psi_k^{(m)}(t) = \dot{x}_k^{(m)}(t) + j\omega_m x_k^{(m)}(t) \triangleq \underbrace{\varphi_k^{(m)}(t)}_{'Slow'\ component} \ \underbrace{e^{j\omega_m t}}_{'Fast'\ component}, \quad j = (-1)^{1/2} \tag{3}$$

where a slow/fast partition of the dynamics in terms of the 'slow' (complex) amplitude $\varphi_k^{(m)}(t)$ and the 'fast' oscillation $e^{j\omega_m t}$ were assumed. Substituting (2) and (3) into (1) and performing multi-phase averaging for each of the 'fast' frequencies gives the *slow flow* of (1)

$$\dot{\underline{\varphi}}_k = \underline{F}_k(\underline{\varphi}_1, ..., \underline{\varphi}_n), \quad \underline{\varphi}_k \in \mathbb{C}^N \tag{4}$$

where $\underline{\varphi}_k = \left\{\varphi_k^{(1)}, ..., \varphi_k^{(N)}\right\}^T$, $k = 1, ..., n$. The number of 'fast' frequencies (N) determines the dimensionality of the slow flow (4).

As an example we consider a weakly damped linear oscillator (LO) coupled to a light attachment by means of essential stiffness nonlinearity of the third degree. In previous work the nonlinear oscillator was termed a 'nonlinear energy sink' (NES) due to its capacity to passively absorb and dissipate energy from the LO over broad frequency ranges.

$$\begin{aligned} \ddot{y} + \omega_0^2 y + \varepsilon\lambda_1 \dot{y} + \varepsilon\lambda_2(\dot{y} - \dot{v}) + C(y - v)^3 = 0 \\ \varepsilon\ddot{v} + \varepsilon\lambda_2(\dot{v} - \dot{y}) + C(v - y)^3 = 0 \end{aligned} \tag{5}$$

This system models the experimental fixture depicted in Figure 1. The strong cubic stiffness nonlinearity is realized by the configuration of Figure 1c. A thin rod (piano wire) *with no pretension* is clamped at both ends, and restricted to move transversely about its center. Assuming that the wire is composed of linearly elastic material, from geometry we get the force – displacement relationship

$$F \approx (EA/L^3)x^3 \equiv Cx^3$$

This explains the strongly nonlinear terms in (5). Such strong geometric nonlinearity occurs when very flexible, thin structural members fixed at their ends undergo transverse (rather than axial) vibrations transverse to their axis.

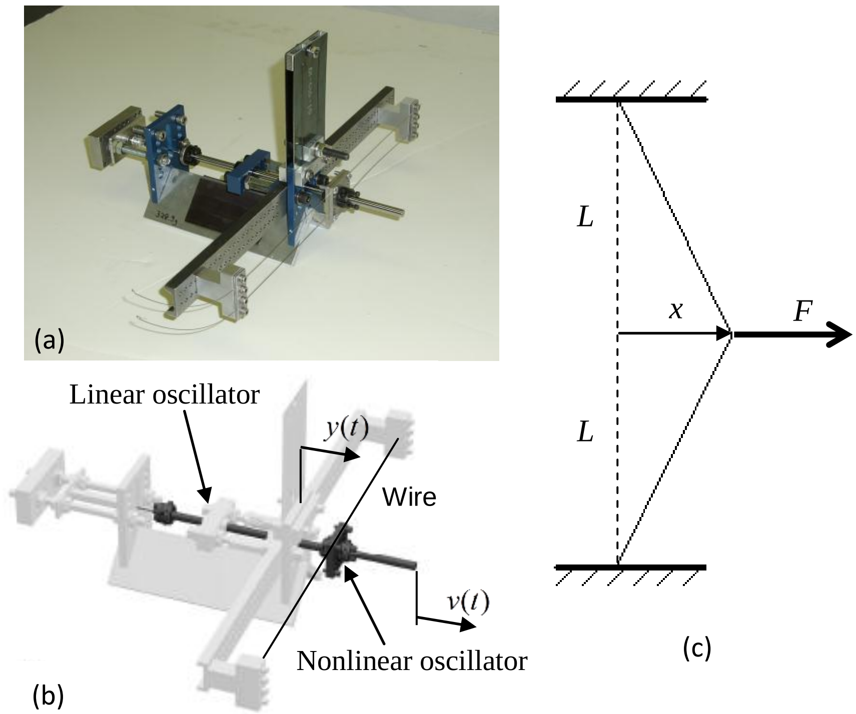


Figure 1. Nonlinear system of coupled oscillators: (a) Experimental fixture, (b) Schematic, (c) Strong geometric nonlinearity.

For (5) we assign $\omega_0 = 1, C = 1, \varepsilon = 0.05, \lambda_{1,2} = 0.03$ and initial conditions $\dot{y}(0) = -0.059, \dot{v}(0) = 0.015, y(0) = v(0) = 0$. Then a 1:3 transient resonance capture (TRC) takes place during which the NES engages in transient resonance with the LO. Figure 2 depicts the responses of the two oscillators in time and frequency (wavelet transform spectra). There exist two dominant 'fast' frequencies in the dynamics, at $\omega_1 = \omega_0$ (high-frequency – HF) and at $\omega_2 = \omega_0 / 3$ (low-frequency – LF), respectively.

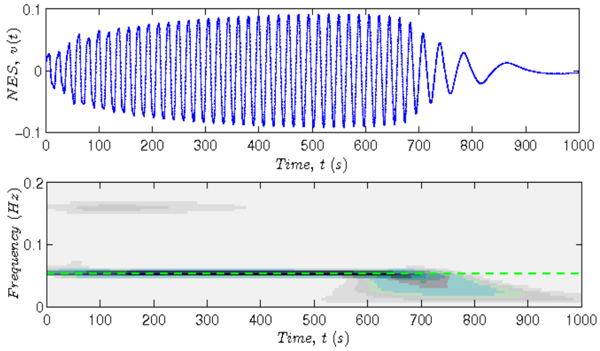
Given that there are only two fast frequencies in the transient responses, we express the responses as $x_1(t) \triangleq y(t) = y^{(1)}(t) + y^{(2)}(t),\ \ x_2(t) \triangleq v(t) = v^{(1)}(t) + v^{(2)}(t)$, and the slow/fast partitions as

$$\begin{aligned} \psi^{(y1)}(t) &\triangleq \dot{y}^{(1)} + j\omega_1 y^{(1)} = \varphi_1^{(1)} e^{j\omega_1 t},\ \ \psi^{(y2)}(t) \triangleq \dot{y}^{(2)} + j\omega_2 y^{(2)} = \varphi_1^{(2)} e^{j\omega_2 t} \\ \psi^{(v1)}(t) &\triangleq \dot{v}^{(1)} + j\omega_1 v^{(1)} = \varphi_2^{(1)} e^{j\omega_1 t},\ \ \psi^{(v2)}(t) \triangleq \dot{v}^{(2)} + j\omega_2 v^{(2)} = \varphi_2^{(2)} e^{j\omega_2 t} \end{aligned} \tag{6}$$

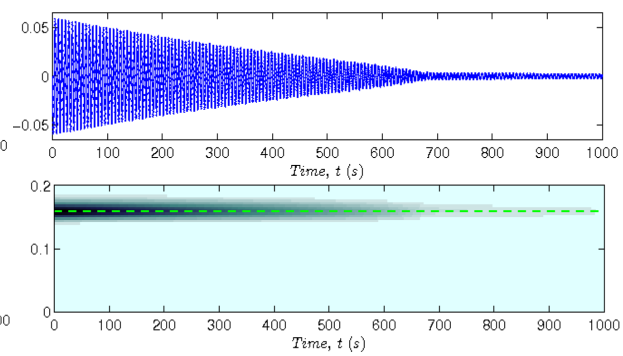
which when substituted in (5) and averaged with respect to ω_1 and ω_2 lead to the slow flow

$$\underline{\dot{\varphi}}_1 = \underline{F}_1(\underline{\varphi}_1, \underline{\varphi}_2), \quad \underline{\dot{\varphi}}_2 = \underline{F}_2(\underline{\varphi}_1, \underline{\varphi}_2) \tag{7}$$

where $\underline{\varphi}_1 = \left\{\varphi_1^{(1)}, \varphi_1^{(2)}\right\}^T$, $\underline{\varphi}_2 = \left\{\varphi_2^{(1)}, \varphi_2^{(2)}\right\}^T$. In Figure 3 we present the slow flow approximation of the NES response, demonstrating that the slow flow accurately approximates the strongly nonlinear transient response.



(a)



(b)

Figure 2. Responses during 1:3 TRC: (a) Nonlinear oscillator (NES), (b) Linear oscillator (LO).

We now show that the slow flow model (4) provides an important theoretical foundation for

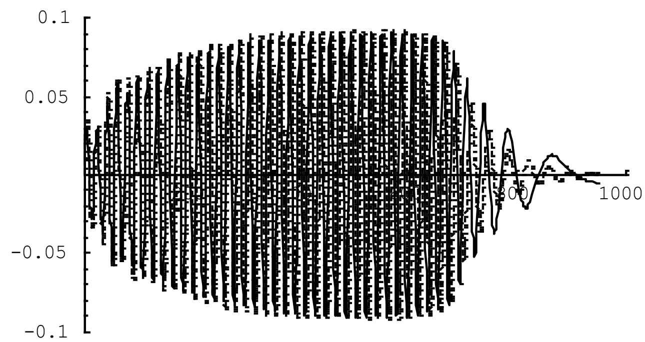


Figure 3. Strongly nonlinear transient response of the NES;
——— exact; -------- slow flow approximation.

performing nonlinear system identification (NSI). To show this we first review the basic elements of *empirical mode decomposition (EMD)*, conceived as a numerical post processing technique for analyzing nonstationary and nonlinear time series. EMD combined with Hilbert spectral analysis has been extensively applied to system identification over the past fifteen years. This decomposition method, based on identifying the characteristic time scales in measured time series, is adaptive, highly efficient, and suitable for nonlinear and nonstationary processes. In particular, EMD yields a complete and nearly (but not completely) orthogonal basis of *intrinsic mode functions – IMFs*; these are oscillatory modes embedded in the time series, each with its own characteristic time scale, whose linear superposition reconstructs the measured time series. In other words EMD is a multi-scale decomposition of a measured time series in terms of embedded oscillatory modes at different time scales.

Until now EMD was applied to time series analysis in an *ad hoc* fashion. The main loop of the EMD algorithm for extracting the IMFs from a signal $x(t)$ consists of the following steps: (i) identify all extrema of $x(t)$; (ii) perform (spline) interpolations of the minima and maxima of $x(t)$, ending up with two envelopes $e\min(t)$ and $e\max(t)$, respectively; (iii) compute the average curve $R(t)=\left[e\min(t)+e\max(t)\right]/2$ (as a residual); (iv) extract the remainder $c(t)=x(t)-R(t)$; (v) apply the previous algorithm on the reminder $c(t)$ until the residual $R(t)$ can be considered as zero-mean under some tolerance (i.e., a stopping criterion). Once this criterion (through the sifting process) is met, the

remainder $c(t)$ is regarded as the effective IMF. By subtracting this IMF from the original time series and applying the algorithm iteratively we extract additional IMFs, so that the original signal $x(t)$ is decomposed sequentially from high- to low-frequency components as

$$x(t) = \sum_{k=1}^{K} c_k(t) + R_{K+1}(t), \; \|R_{K+1}(t)\| << tol \tag{8}$$

By this process the superposition of the K leading IMFs reconstruct approximately the measured time series; however, due to the *ad hoc* nature of the sifting algorithm only a subset of these IMFs are physically meaningful with the rest being of spurious nature. The *dominant* (and physically meaningful) IMFs can be identified by comparing their instantaneous frequencies to the wavelet transform (WT) spectra of the original time series; the instantaneous frequencies of the dominant IMFs coincide with the dominant harmonics of the wavelet spectra. This process also identifies the dominant time scales (or frequencies) of the dynamics in the time series. It follows that EMD provides the following *ad hoc* numerical decomposition of the dynamics of the general system (1),

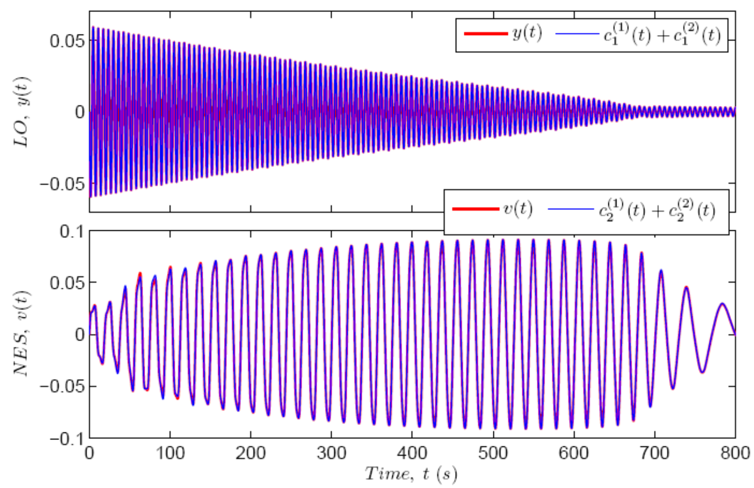


Figure 4. Reconstruction of time series in (5) with IMFs.

$$x_k(t) = c_k^{(1)}(t) + ... + c_k^{(N)}(t), \quad k = 1,...,n \tag{9}$$

where $c_k^{(m)}(t)$ is the $m-th$ dominant IMF of the response $x_k(t)$, associated with the fast frequency ω_m, with $\omega_1 > ... > \omega_N$. Comparing the EMD result (9) with the slow/fast decomposition (2) we clearly establish the possibility of relating the two expansions *thus providing a theoretical basis for EMD in terms of the underlying slow flow dynamics.* This important result was established in and will form the basis of our NSI method. Some preliminary remarks, however, are appropriate. In Figure 4 we reconstruct the responses of system (5) using two dominant IMFs for each subsystem, namely $c_1^{(1)}(t)$ ($HF\ LO$ component at ω_1) and $c_1^{(2)}(t)$ ($LF\ LO$ component at ω_2 – it turns out that this is negligible) for the LO, and two dominant IMFs $c_2^{(1)}(t)$ ($HF\ NES$ component) and $c_2^{(2)}(t)$ ($LF\ NES$ component) for the NES. By comparing with the exact numerical time series we establish the completeness of the decomposition.

Now, the Hilbert transform (HT) $h(t)$ of a (mono-component) signal $y(t)$ is computed as

$$h(t) \equiv \mathcal{H}\left[y(t)\right] = \frac{PV}{\pi} \int_{-\infty}^{+\infty} \frac{y(s)}{t-s} ds \quad (10$$

where PV stands for Cauchy principal value. Moreover, a complex function whose imaginary part is the Hilbert transform of the real part is analytic. Motivated by this result we may complexify the $m-th$ IMF $c_k^{(m)}(t)$ of the time series $x_k(t)$ in (9) by defining the analytic complex function,

$$\hat{\psi}_k^{(m)}(t) \equiv c_k^{(m)}(t) + j\,\mathcal{H}\left[c_k^{(m)}(t)\right] \quad (11)$$

where $j = (-1)^{1/2}$. This enables the computation of the *instantaneous amplitude and phase of the* $m-th$ *IMF* as

$$\hat{A}_k^{(m)}(t) = \left\{ c_k^{(m)2}(t) + \mathcal{H}\left[c_k^{(m)}(t)\right]^2 \right\}^{1/2}, \quad \tan\hat{\theta}_k^{(m)}(t) = \mathcal{H}\left[c_k^{(m)}(t)\right] / c_k^{(m)}(t) \quad (12)$$

from which the *instantaneous frequency* of the IMF is computed as $\hat{\omega}_k^{(m)}(t) = \dot{\hat{\theta}}_k^{(m)}(t)$. This leads to the following slow-fast representation of the complexified IMF (11):

$$\hat{\psi}_k^{(m)}(t) \triangleq \hat{A}_k^{(m)}(t)\, e^{j\hat{\theta}_k^{(m)}(t)} = \underbrace{\hat{A}_k^{(m)}(t) e^{j\left[\hat{\theta}_k^{(m)}(t) - \omega_k^{(m)} t\right]}}_{'Slow'\ component} \quad \underbrace{e^{j\omega_k^{(m)} t}}_{'Fast'\ component} \quad (13)$$

We note that *by complexifying the identified IMFs in (12) we have a direct way to relate them to the governing slow flow dynamics* [e.g., the complex amplitudes $\psi_k^{(m)}(t)$ in (3)]. This will provide a way to physically interpret the dominant IMFs in terms of the slow flow dynamics.

Although EMD combined with the Hilbert transform (HT) forms a powerful post-processing tool for extracting intrinsic oscillating components from a time series and identifying the dominant time scales of the dynamics, it has some important deficiencies. First, application of EMD may lead to *spurious IMFs*, so that physically meaningful results from EMD can only be obtained if these are omitted from further consideration in the analysis; spurious IMFs are the direct result of the well established lack of orthogonality of the IMFs. *The deletion of spurious IMFs from further consideration can be performed by comparing the instantaneous frequencies of the IMFs to the WT spectra of the time series and eliminating nonphysical IMFs from further consideration.* Hence, we can identify a set of *dominant IMFs* from the measured time series. Second, there are concerns regarding the *frequency resolution* of the EMD results. Indeed, in order to obtain meaningful results when applying HT to the IMFs, it is necessary that these are mono-component or, at least, narrowband (otherwise mixed-mode IMFs in the form of beat phenomena are obtained possessing closely spaced frequencies). Finally, there are issues concerning the *uniqueness* of the EMD results. EMD does not result in a unique decomposition of a measured time series since it is applied in an *ad hoc* manner and depends on a free stopping parameter; that is, EMD is not robust in practice. The set of extracted IMFs can be considered as a basis for reconstructing the original measured time series if it satisfies (or nearly satisfies) the basic conditions of *completeness* and *orthogonality*. By virtue of the EMD sifting algorithm, completeness of the IMFs is guaranteed by construction. It is the lack of orthogonality between IMFs, however, that generates spurious features in the results and prevents uniqueness of the decomposition. These issues have been addressed through the use of masking and mirror-image signals that lead to well-decomposed, nearly orthogonal sets of IMFs.

We can now relate the theoretical slow flow decomposition (2) and the numerically derived dominant measured IMFs (9), a major outcome of this work. The response of the $k-th$ DOF of (1) can be expressed as

Slow flow: $x_k(t) = x_k^{(1)}(t) + ... + x_k^{(N)}(t),\ \psi_k^{(m)}(t) = \dot{x}_k^{(m)}(t) + j\omega_m x_k^{(m)}(t) \triangleq \underbrace{\varphi_k^{(m)}(t)}_{\text{'Slow' component}} \quad \underbrace{e^{j\omega_m t}}_{\text{'Fast' component}}$

EMD: $x_k(t) = c_k^{(1)}(t) + ... + c_k^{(N)}(t),\ \hat{\psi}_k^{(m)}(t) \equiv c_k^{(m)}(t) + j\mathcal{H}\left[c_k^{(m)}(t)\right] = \underbrace{\hat{A}_k^{(m)}(t)e^{j\left[\hat{\theta}_k^{(m)}(t)-\omega_k^{(m)}t\right]}}_{\text{'Slow' component}} \quad \underbrace{e^{j\omega_k^{(m)}t}}_{\text{'Fast' component}}$

Given, however, that the time series is decomposed in terms of dominant IMFs, it follows that $\omega_k^{(m)} \approx \omega_m$ for $m = 1,..., N$, where N is the number of dominant harmonic components in the slow flow decomposition of the dynamics (i.e., the dimensionality, or the number of significant frequency-time scales in the dynamics). It follows that the *above partitions can be related since they represent identical theoretical and numerical multi-scale slow-fast decompositions of the measured time series*

$$\underbrace{x_k^{(m)}(t)}_{\textit{Theoretical}\ \text{model}} \quad \to \quad \underbrace{c_k^{(m)}(t)}_{\textit{Numerical}\ \text{model}} \quad , \qquad k = 1,...,n, \quad m = 1,..., N \tag{14a}$$

Thus, we make the association

$$\psi_k^{(m)}(t) \to \hat{\psi}_k^{(m)}(t) \ \Rightarrow\ \underbrace{\varphi_k^{(m)}(t) \to \hat{A}_k^{(m)}(t)e^{j\left[\hat{\theta}_k^{(m)}(t)-\omega_m t\right]}}_{\text{Equivalence of 'slow' complex amplitudes}} \tag{14b}$$

These relations provide a physics-based theoretical foundation for EMD, whereby the dominant IMFs represent the underlying slow flow and, hence, capture all the important (multi-scale) dynamics.

This important result provides the foundation for the development of our new NSI method. We note that no assumptions have been made regarding the type or dimensionality of system and the type and strength of the nonlinearity. It follows that the slow-fast partitions discussed above and the physical interpretation of the results of EMD should hold for a broad class of dynamical systems. In fact, we have demonstrated that EMD can provide us with the added benefit of separating the smooth from non-smooth dynamics by localizing all non-smooth effects (e.g., due to clearances, backlash, etc.) in the leading order IMFs. Hence, our NSI methodology can be extended to non-smooth dynamical systems.

An example of this correspondence is now given in terms of our previous strongly nonlinear example (5). In Figures 5a,c we depict a comparison between the magnitudes of the slowly varying amplitudes of the low- and high-frequency (LF and HF) components of the LO and the NES derived by slow flow analysis and EMD; in Figures 5b,d we present the corresponding slow components in the complex plane (which incorporates also phase information). There is good correspondence between the analytical and numerical results for these two dominant components (cf. Fig.2) confirming the previous theoretical arguments.

Nonlinear System Identification: Global and Local Issues

Compared with linear modal analysis, NSI of dynamical systems is far more complex. Nonlinear systems are energy– and initial conditions–dependent, so that even the simple task of identifying a set of (linearized) modal matrices modified ('perturbed') by nonlinear corrections might be an oversimplification of the problem. Using as an example the 1:3 transient resonance capture of the strongly nonlinear system (5) we should recognize that *a change in initial conditions can result in drastically*

different transient dynamics. To illustrate this point we introduce in our discussion the concept of *frequency-energy plot (FEP)* , which provides a *global* picture of the dynamics.

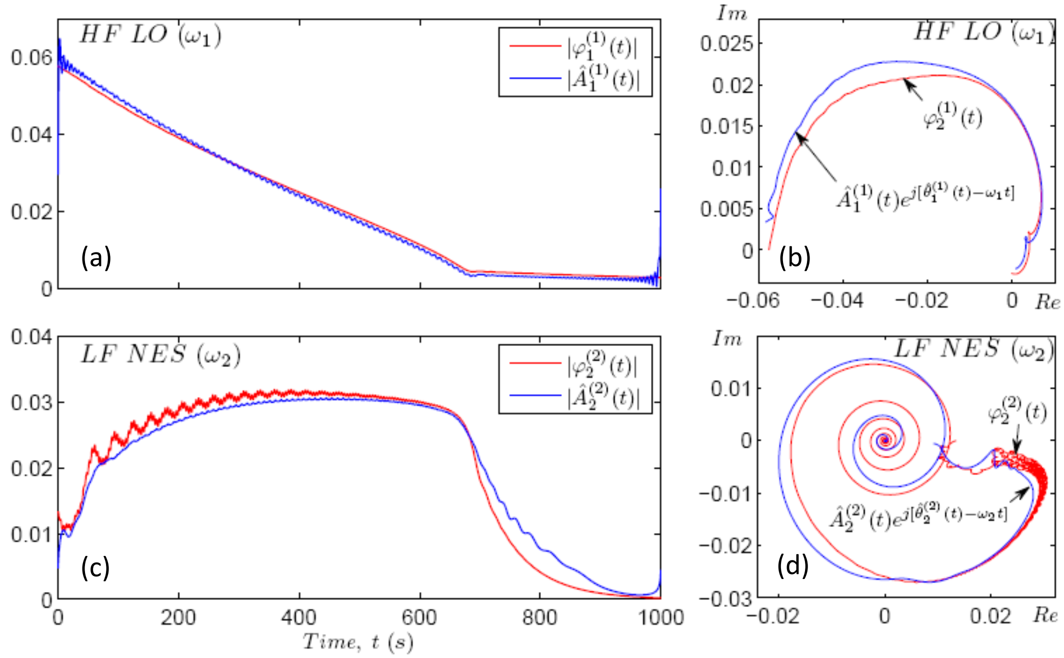
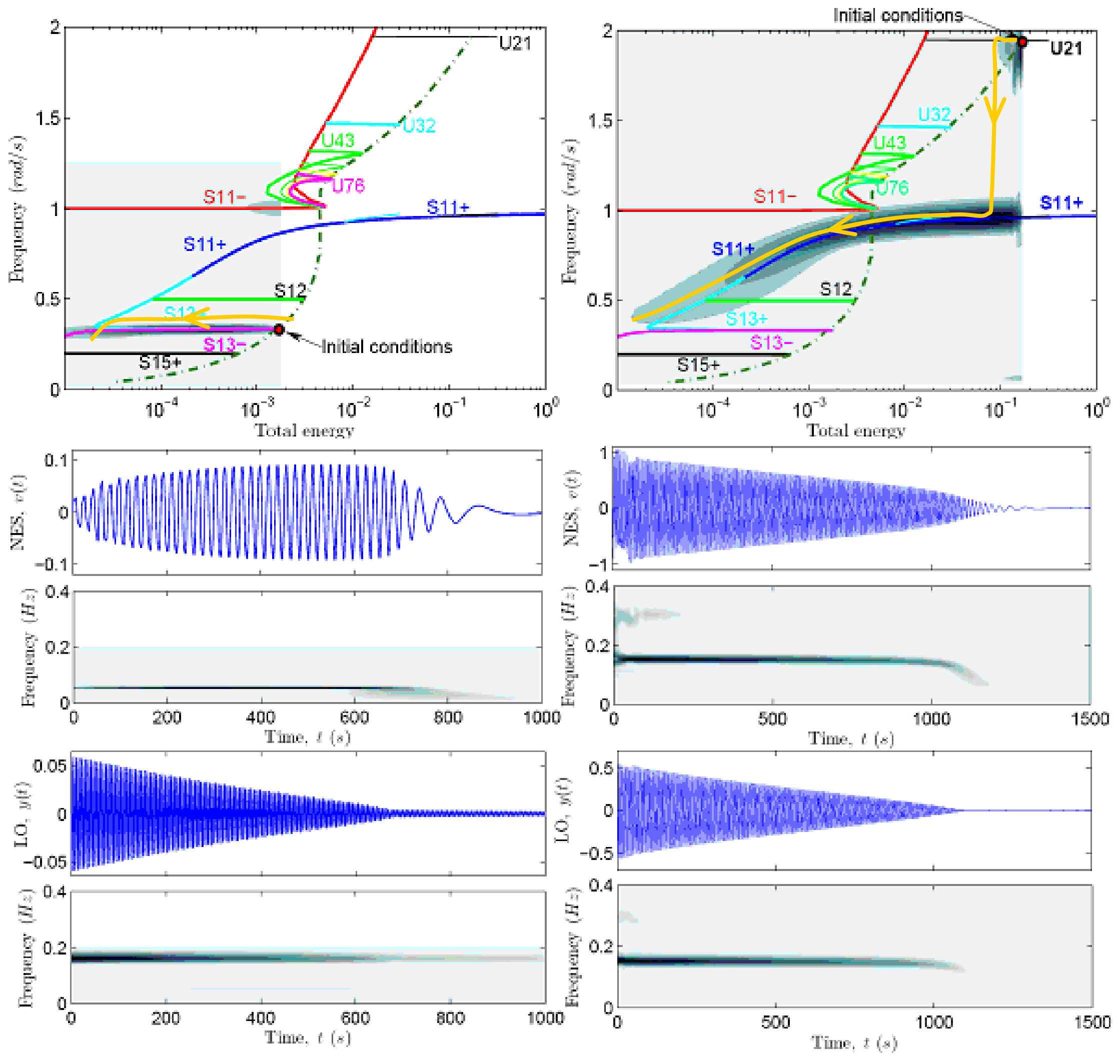


Figure 5. Correspondence between theory (slow flow dynamics) and numerics (EMD results) for the strongly nonlinear system (5): (a,b) HF response of LO (ω_1), (c,d) LF response of the NES (ω_2).

To construct the Hamiltonian FEP for system (5) we set $\lambda_1 = \lambda_2 = 0$ and compute its periodic orbits. The resulting FEP is in the background in Figures 6a,b, where a frequency index (FI) of a periodic orbit is depicted against its (conserved) energy. Symmetric periodic orbits $Snm\pm$ correspond to curves in the configuration plane (y,v), with $(m:n)$ indicating the internal resonance realized (e.g., a 1:1 internal resonance is realized on $S11\pm$, with both the LO and the NES oscillating with identical dominant frequencies). The ($\pm$) signs indicate in-phase or out-of-phase motions of the LO and the NES. Unsymmetric periodic orbits $Upq\pm$ are Lissajous curves in the configuration plane. Then, the FI of a periodic orbit on branches $Snm\pm$ and $Unm\pm$ is given by $FI = n\omega_0 / m$. The (seemingly simple) system (5) possesses a countable infinity of periodic orbits! *Near-horizontal branches are linearized or weakly nonlinear motions* (weak frequency – energy dependence), whereas *curved branches imply strongly nonlinear responses* (the corresponding linear two – DOF system would possess a FEP with just two horizontal lines corresponding to its natural frequencies!).

A very useful feature of the Hamiltonian FEP is its relation to the transient dynamics of the corresponding weakly damped system. As discussed in the dynamics of the weakly damped system can be closely related to the underlying Hamiltonian dynamics: indeed, for weak damping the transient dynamics tracks specific branches of periodic orbits in the FEP. As energy decreases due to damping dissipation sudden transitions may occur as the damped response jumps from the neighborhood of one branch of periodic solutions to another. The sequence of branches of periodic orbits tracked by the damped dynamics in the FEP is ultimately dictated by the initial conditions and the level of damping in the system. Using as an example system (5), the close correspondence between the weakly damped and Hamiltonian dynamics can best be demonstrated by superimposing on the Hamiltonian FEP the wavelet spectra of the time series corresponding to the difference $y(t) - v(t)$. This is done in Figure 6a for the 1:3 transient resonance capture depicted of Figure 2.



(a) (b)

Figure 6. Two damped nonlinear transitions of system (5) depicted in the FEP: (a) 1:3 Transient resonance capture, (b) transition with initial conditions $y(0)=v(0)=\dot{v}(0)=0,\ \dot{y}(0)=-0.579$ *; yellow lines indicate schematically the damped transitions.*

In the same figure we depict the transient responses $v(t)$ and $y(t)$ together with their wavelet spectra. We see that during 1:3 transient resonance capture the dynamics tracks closely the 1:3 subharmonic 'tongue'), so a relatively simple topological picture of the transition emerges. Using, however, a different set of initial conditions we get drastically different dynamics as evidenced by the high-frequency transition depicted in Figure 6b. In this case the dynamics is initiated on the higher energy superharmonic tongue $U21$ so the damped dynamics tracks a completely different set of branches in the FEP. This results in a complicated multi-frequency nonlinear transition which, however, can be analyzed through theoretical and numerical slow-fast partitioning of the dynamics. It is evident though that *performing NSI based only on either one of the measured time series we would miss a component of the dynamics*. Moreover, even if both transitions are analyzed, NSI would still be incomplete as there would still be dynamics not captured by the transitions of Figure 6.

The previous example highlights the important challenges that the analyst is faced with when performing NSI. *The first challenge is to address the (generic) feature of nonlinear systems to exhibit qualitatively different responses with varying energy and/or initial conditions*. To address this challenge one needs to adopt a *global approach* for identifying the basic (essential) dynamical features of a system over broad frequency and energy ranges. The second challenge is to be able to identify complex multi-frequency transitions (such as the ones depicted in Figure 6) for fixed sets of initial conditions (or energy). This dictates a *local approach* to NSI whereby, a specific nonlinear transition is considered and the task is to identify the nonlinear modal interactions that govern this transition. The NSI methodology that we have developed promises to address both of the above challenges by introducing a combined global / local approach to NSI: global features of the dynamics are identified in the frequency-energy domain by constructing FEPs, whereas local transitions (such as the ones depicted in Figure 6) are identified by constructing appropriate slow-flow models. In this way we ensure that both the global and local requirements of NSI are addressed. The added benefit of our approach is that it is based on direct analysis of measured time series which contain complete information of the nonlinear dynamics to be identified.

Archival Publications Referencing the Current Grant

Below is the sequence of archival publications with results funded in part by the AFOSR grant FA9550-07-1-0335, outlining the development of the nonlinear system identification method.

"Physics-Based Foundation for Empirical Mode Decomposition: Correspondence Between Intrinsic Mode Functions and Slow Flows," 2009, Y. S. Lee, S. Tsakirtzis, A. F. Vakakis, D. M. McFarland, L. A. Bergman. AIAA Journal, Vol. 47, No. 12, pp. 2938-2963.
This paper developed the preliminary theoretical framework for the NSI method. We studied the correspondence between analytical slow flows (derived from complexification-averaging) and empirical slow-flows (composed of the IMFs derived from EMD). We showed equivalence between the theoretical and computational slow flows which is at the heart of our NSI approach, and provides a physics-based theoretical foundation for EMD, which until then was performed in an ad hoc fashion. We provided a rigorous mathematical foundation for this equivalence, and provided demonstrations with certain nonlinear dynamical systems.

"A Time-Domain Nonlinear System Identification Method Based on Slow-Fast Dynamic Partitions," Y. S. Lee, S. Tsakirtzis, A. F. Vakakis, L. A. Bergman, D. M. McFarland, Meccanica, (in press). DOI: 10.1007/s11012-010-9327-7.
In this paper we develop the NSI method. Based on the theoretical foundation for empirical mode decomposition developed in 1, we developed the NSI method based on multiscale dynamic partitions and direct analysis of measured time series, with no presumptions regarding the type and strength of the system nonlinearity. Hence, the method is developed for a broad class of time-variant/time-invariant, linear/nonlinear, and smooth/non-smooth dynamical systems. We show how the method provides in a systematic way reduced order models of strongly nonlinear transitions in the form of coupled or uncoupled oscillators with time-varying or time-invariant coefficients forced by nonhomogeneous terms representing nonlinear modal interactions. Hence, the method not only identifies the basic (dominant) time scales of the dynamics, but also the time histories of the nonlinear interactions across the scales of the dynamics. We provided examples of the NSI method by analyzing strongly nonlinear modal interactions in dynamical systems with essentially nonlinear attachments.

"A Global-Local Approach to System Identification: A Review," 2010, Y. S. Lee, A. F. Vakakis, D. M. McFarland, L. A. Bergman, Structural Control and Health Monitoring, Vol. 17, pp. 742-760. DOI: 10.1002/stc.414. Invited paper in Honor of Prof. George W. Housner.

In this paper we discuss the global/local duality of the NSI method, dictated by the well known sensitivity of dynamical systems to initial and forcing conditions. This is a unique feature of our NSI method, since it fully accounts for global as well as local identification of the different types of dynamics that a nonlinear system can realize. We present a review of a time-domain (NSI) method to study the local and global dynamics of mechanical systems. In addition, we demonstrate that the NSI method is capable of partitioning smooth from non-smooth effects in the dynamics of oscillators with discontinuous nonlinearities, such as vibro-impacts. In particular, we show that by "removing" non-smooth (vibro-impact) effects from measured time series we are able (i) to focus in the identification of the underlying smooth components of the dynamics, and (ii) to identify interesting patterns in the non-smooth components of the dynamics, e.g., patterns of vibro-impacts in the vibro-impacting system. This demonstrates that our NSI method is applicable to systems with `hard' (discontinuous) nonlinearities where traditional methods are inapplicable.

"Nonlinear System Identification of the Dynamics of an Elastic Rod with an Essentially Nonlinear Attachment," 2010, S. Tsakartzis, Y. S. Lee, A. F. Vakakis, L. A. Bergman, D. M. McFarland, Communications in Nonlinear Science and Numerical Simulation, Vol.15, pp. 2617-2633. DOI: 10.1016/j.cnsns.2009.10.014.
In this paper we apply the NSI method to identify strongly nonlinear interactions in the dynamics of a linear elastic rod with an essentially nonlinear attachment at its end. We are able to construct a reduced order model that fully reconstructs a multi-frequency transition in the dynamics. This paper represents a first application of the NSI method to the transient dynamics of a linear elastic continuum with a strongly nonlinear (nonlinearizable) attachment.

"Nonlinear System Identification of the Dynamics of Aeroelastic Instability Suppression Based on Targeted Energy Transfers," Y. S. Lee, A. F. Vakakis, D. M. McFarland, L. A. Bergman, 2010, Aeronautical Journal of the Royal Aeronautical Society, Vol. 114, No. 1152, pp. 61-82.
In this paper we apply the NSI method to identify the nonlinear modal interactions that occur during aeroelastic flutter suppression due to targeted energy transfer (TET) from an oscillating wing to an essentially nonlinear attachment. We construct reduced order models in the form of nonlinear interaction models (NIMs) that not only provide information on modal energy exchanges under nonlinear resonant interactions, but also can be used to study the robustness of the aeroelastic instability suppression. Moreover, we discuss the usefulness of NIMs in constructing frequency-energy plots that reveal global features of the dynamics to distinguish between different TET mechanisms and to study robustness of aeroelastic instability suppression.

Student and Post-doctoral Researchers Supported in Part by the Current Grant

Post-doc: Young S. Lee, PhD (UIUC, 2006). Now assistant professor of mechanical and aerospace engineering, New Mexico State University, Las Cruces, NM.

Student: Sean A. Hubbard, MS (UIUC, 2009). Now doctoral student in the aerospace engineering department, UIUC.

Student: Mercedes Mane, MS (UIUC, 2011). Now laboratory assistant, aerospace engineering department, UIUC.

Changes in research objectives, if any:

None

Change in AFOSR program manager, if any:

Dr. Victor Giurgiutiu, followed by Dr. David Stargel

Extensions granted or milestones slipped, if any:

One six month no-cost extension granted.

New discoveries, inventions, or patent disclosures:

Physical basis for empirical mode decomposition (EMD) in dynamical systems documented.
No inventions or patent disclosures.